CIRCUIT CELLAR ONLINE

FEATURE ARTICLE

Hristo Stefanov

Determining Measurement Accuracy

To measure different signals, yu've got to define the terms of measurement. Most of us use the term "error" in assessing uncertainties. Because every measuremnet has some amount of error, there is always inaccuracy, which can be attributed to many factors. In this article, Hristo takes us through through a series of different types of errors, from random to systematic.

the vast invasion of embedded systems in the real world engenders the need of measuring different signals. Even a battery charger needs to measure the temperature of the cell. But if you measure a signal without knowing the error, you are doing nothing, because every measurement is done with some amount of error. Someone can tell you that the ambient humidity is 70% RH. From this message, you don't get any information about the accuracy. Imagine taking a measurement with a horse filament!

Usually you would say that the ambient humidity is 70% RH with maximum error of ±2% RH. Even the error cannot be specified without error. So the most correct way is to say that at 95% probability, the ambient humidity is in the range 70% RH ±2% RH. This way, you get to know the uncertainty of the measurement.

The uncertainty is a figure of merit associated with the actual measured value, the boundary limits within which the true value lies. Contributors to this potential for inaccuracy include the performance of the equipment used to make the measurement, the test process or technique itself, and environmental effects. The assessment of uncertainties of measurement is a task more suited to a mathematician rather than the average engineer. That is why most of us use the term error. By definition, errors are known and can usually be taken into account by correcting measured values, whereas uncertain measurements merely define the limits of potential inaccuracy.

DEFINING THE TERMS

I'd like to review some definitions that I will be using in this article. That way you'll receive a better understanding of the content.

Accuracy of measurement is the closeness of the agreement between the result of a measurement and a true value of the measured value. Note that

Parameter	Symbol	Value
Input offset voltage	V_{OS}	10 µV
Long-term input offset voltage stability	ΔV_{OS}/time	0.2 µV/mo.
Input offset current	I_{OS}	1 nA
Input bias current	I_B	1.5 nA
Power supply rejection ratio	PSSR	120 dB
Average input offset voltage drift	TCV_{OS}	0.1 µV/°C
Average input offset current drift	TCI_{OS}	25 pA/°C
Average input bias current drift	TCI_B	25 pA/°C
Large-signal voltage gain	A_{VO}	2000 V/mV

Table 1—*Here you can see the electrical characteristics of the OP177 op-amp.*

Error source	Expression for the absolute error	Absolute error	Error as ppm of full scale
Offset errors	RTI	RTI	
Input offset voltage	10 µV	10 µV	100 ppm
Input offset current	1 nA × (1 kilohm || 99 kilohm)	0.99 µV	9.9 ppm
Input bias current	1.5 nA × (1 kilohm || 99 kilohm)	1.485 µV	14.85 ppm
PSSR	125 dB = 0.56 µV/V	0.56 µV/V	5.6 ppm
Input offset voltage drift	0.1 µV/°C	0.1 µV/°C	1 ppm/°C
Input offset current drift	25 pA/°C	0.025 µV/°C	0.25 ppm/°C
Input bias current drift	25 pA/°C	0.025 µV/°C	0.25 ppm/°C
Input offset voltage stability	0.2 µV/mo.	0.2 µV/mo.	2 ppm/mo.
Total offset error	130.35 ppm + 1.5 ppm/°C + 2 ppm/mo.		
Gain errors	RTO	RTO	
Finite gain of op-amp	G × $Vout_{MAX}$/A = 100 × 10 V/2000V/mV	0.5 mV	50 ppm
R1 tolerance	ΔG × Vin_{MAX} = (99 kilohm/(1 kilohm × 1 kilohm)) × 1% × 1 kilohm × 100 mV	99 mV	9900 ppm
R2 tolerance	ΔG × Vin_{MAX} = (1/1 kilohm) × 1% × 99 kilohm	99 mV	9900 ppm
R1 drift	ΔG × Vin_{MAX} = (99 kilohm/(1 kilohm × 1 kilohm)) × 50 ppm × 1 kilohm/°C × 100 mV	0.495 mV/°C	49.5 ppm/°C
R2 drift	ΔG × Vin_{MAX} = (1/1 kilohm) × 50 ppm × 99 kilohm/°C × 100 mV	0.495 mV/°C	49.5 ppm/°C
Total gain error	19850 ppm + 99 ppm/°C		

Table 2

accuracy is a qualitative concept, and the term "precision" should not be substituted for "accuracy."

Error is the difference between a computed, estimated, or measured value and the true, specified, or theoretically correct value. It can also be described as a deviation from a correct value caused by a malfunction in a system or a functional unit.

Random error is the result of a measurement minus the mean that would result from an infinite number of measurements of the same measured value carried out under repeatable conditions. Note that random error is equal to error minus systematic error, and because only a finite number of measurements can be made, it's possible to determine only an estimate of random error.

And finally, systematic error is the mean that would result from an infinite number of measurements of the same measured value carried out under repeatable conditions minus the true value of the measured value. Note that systematic error is equal to error minus random error, and like true value, systematic error and its causes cannot be completely known.

Hereafter, I will talk about only systematic errors because random errors are difficult to assess.

A LITTLE THEORY

Every measuring system consists of function blocks in which signal conversion takes place. These blocks can be generally called measuring converters, or simply converters. An amplifier, ADC, filters, and so on are such converters. In the ideal converter, the output variable Y is a function of the input variable X only:

$$Y = f(X)$$

In any real converter, the transfer function differs from the ideal one because of the presence of sources that cause inaccuracy (error sources):

$$Y = f_R(X, A_1 \ldots A_n)$$

If you substitute zero for all sources $A_1 \ldots A_n$, you obtain the transfer function of the ideal converter:

$$Y = f_R(X, 0, \ldots 0)$$

The absolute error in the real converter is given by:

$$Y_{EER} = f_R(X, A1 \ldots An) - f_R(X, 0 \ldots 0) = DY$$

The total differential of the function f_R is:

$$dY = \frac{\partial f_R}{\partial X} dX + \frac{\partial f_R}{\partial A_1} dA_1 + \ldots + \frac{\partial f_R}{\partial A_n} dA_n$$

Consequently, for infinitesimal values of $A_1 \ldots A_n$, you can write:

$$Y_{ERR} = \frac{\partial f_R}{\partial A_1} dA_1 + \ldots + \frac{\partial f_R}{\partial A_n} dA_n = \sum_{i=1}^{n} \frac{\partial f_R}{\partial A_i} dA_i$$

This equation provides the justification for representing the overall systematic error of the linear converter as the sum of error terms. Every term corresponds to one source of error. This approach is the foundation for the error budget method.

SOURCES OF ERROR

I will not classify the sources of error, nor will I try to list all of them. I simply want to clarify some potential risks. There is an infinite number of error sources that degrade the performance of a measuring system, so you have to select and consider the most significant. This is not a trivial problem and will need some intuition.

Information for primary errors can be obtained from datasheets and reference books. They result from the imperfection of components and methods. Hidden errors come from sources that are not obvious, and most of them appear only at performance test. It is difficult to identify and assess such errors at design time, but it is worth making the same effort.

The following is my "top-10" list of hidden sources of error:

- power supply instability
- interference
- self-heating

- thermoelectric effect
- PCB deformation
- moisture absorption in the PCB
- ground plane voltage drops
- parasitic resistance, capacitance, and inductance in the PCB
- contamination
- dielectric absorption

ERROR BUDGET

Error budget is a systematic approach to analytical determination of overall accuracy at design time. As you saw previously, there is reason to consider the overall systematic error as the sum of error terms because every term corresponds to one source of error. Therefore, you can build a budget where every article corresponds to one error term, hence error budget. Every article includes the source of error, the expression that assesses the error, and the evaluation of the expression. You can summarize the building stages as follows:

- collecting information about sources of error
- calculating to assess the influence of components' characteristics on circuit performance
- filling up the budget table
- evaluating the totals
- assigning the deviation of factors that affect sources of error (such factors are temperature, time, etc.).
- evaluating the final overall error
- analyzing the results

To simplify the calculations, assume that the error terms are small and not correlated. Because most of the error terms are bipolar (and not correlated), evaluate every error and total by modulus.

Let's examine the linear converter (because of its common usage) and derive the most important equations needed to assess its accuracy.

The transfer function is given by:

$$Y = GX$$

where G = gain, X = input value, and Y = output value.

In the real transfer function, there is deviation in the gain and zero:

$$Y_R = (G + DG)X + Y_0$$

where DG = gain deviation and Y_0 = zero deviation (offset).

The absolute error () is given by:

$$Y_{EER} = Y_R - Y$$
$$Y_{EER} = XDG + Y_0$$

where XDG = gain error and Y_0 = offset error.

You can see that the absolute error of the linear converter is the sum of two terms, gain error and offset error. Furthermore, the gain error is dependent on X and the offset error is independent of X. Maximum absolute error occurs when the value of X is maximum (X_{MAX}):

$$Y_{ERRMAX} = X_{MAX}\Delta G + Y_0$$

Let's assume that the input span (input scale) is 0 – X_{MAX} and the output span (output scale) is 0 – Y_{MAX}. You can express the maximum absolute error as a fraction of the full output scale range Y_{MAX}:

$$ERR_{FS} = \frac{Y_{ERRMAX}}{Y_{MAX}} = \frac{X_{MAX}\Delta G}{Y_{MAX}} + \frac{\Delta Y_0}{Y_{MAX}} = \frac{\Delta G}{G} + \frac{Y_0}{Y_{MAX}}$$

Because:

$$\frac{Y_{MAX}}{X_{MAX}} = G$$

the gain error can be expressed independently of the full-scale range, and the offset error:

$$\frac{Y_0}{Y_{MAX}}$$

is expressed as a fraction of the full-scale range.

Fractional errors can be converted to ppm (parts per million) by multiplying by 1×10^6 and to percent by multiplying by 100.

In the linear converters, absolute errors can be referred to the input (RTI) or output (RTO). You can convert RTI errors to RTO by multiplying by gain (G). Errors expressed as a fraction (of the full-scale range) are independent of the reference. Therefore, you can sum fractional errors obtained from RTI and RTO absolute errors.

In the following example, I will derive expressions for RTI and RTO absolute errors and evaluate them. And, I will express the obtained absolute errors as fractional ones and calculate the totals separating gain and offset errors.

ERROR BUDGET EXAMPLE

The well-known noninverting amplifier based on a single op-amp is used to demonstrate the error budget approach (see Figure 1).

circuit introduces too many sources of error.

The characteristics of the amplifier are:

- maximum input voltage: $V_{INMAX} = 100\ mV$
- maximum output voltage: $V_{OUTMAX} = 10 V$
- nominal gain: G = 100
- transfer function: $V_{OUT} = GV_{IN}$

There's an infinite number of error sources that degrade the circuit performance. Take into account those that cause considerable error. The primary sources of errors come from op-amps and resistors in the feedback. You can get the data required for building the error budget from the components' worst-case specifications.

The electrical characteristics of the op-amp (the OP177 from Analog Devices) can be seen in Table 1. For the resistors, the temperature coefficient is ±50 ppm and the tolerance is ±1%.

OFFSET AND GAIN ERRORS

Offset errors in the amplifier circuit are caused mainly by op-amp input offset voltage and input currents. All absolute offset errors listed in Table 1 are referred to the input (RTI).

Factors causing gain errors in the

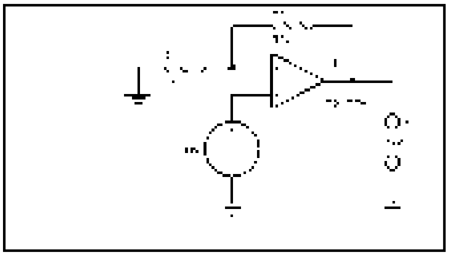

Figure 1—*This simple amplifier circuit introduces too many sources of error.*

amplifier circuit come from the resistors' value deviation and the finite open-loop gain of the op-amp.

The gain of the circuit (assuming infinite open-loop gain) is:

$$G = 1 + \frac{R_2}{R_1}$$

To estimate the influence of resistor deviation on the gain, you have to differentiate the above equation with respect to R_1 and R_2:

$$dG = \frac{1}{R_1} dR_2$$

$$dG = -\frac{R_2}{R_1^2} dR_1$$

Let's use these relations in the error budget to calculate the error caused by the resistors' tolerance and temperature coefficient.

To estimate the error caused by the finite open-loop gain (A_{VO}) of the op-amp, I will include in the error budget an article that is the voltage between the two op-amp inputs. This voltage is:

$$\frac{V_{OUT}}{A_{VO}}$$

Using these assumptions, I've completed the error budget table (see Table 2 <LINK>). Assigned are the deviation of factors that affect some sources of error—time (12 months), power supply voltage (±1 V), and temperature drift (± 25°C):

Offset error = 130.35 ppm + 1.5 ppm/°C + 2 ppm/mo.= 130.35 ppm + 1.5 ppm/°C x 25°C + 2 ppm/mo. x 12 mo. = 130.35 + 37.5 + 24 = 191.85 ppm of full scale
Gain error = 19,850 ppm + 99 ppm/°C = 19,850 ppm + 99 ppm/°C x 25°C= 19,850 + 2475 = 22,325 ppm of full scale
Overall error = 191.85ppm + 22,325ppm = 22,516.85 ppm of full scale

The next logical step after calculating the errors is to analyze them and see how they can be reduced. From the above results, you can see that an analyzed circuit is not quite accurate as a result of the large gain error, 22325 ppm of full scale. The main reason for this is the tolerance of the resistors R_1 and R_2; it causes 19,800-ppm gain error. To avoid this significant source of error, you can implement trimming either R_1 or R_2. This approach can reduce the overall error to 2716.85 ppm! Furthermore, you can reduce gain error by usage of resistors with matched temperature coefficients so that they track.

The most significant factor causing offset error is the input offset voltage of the op-amp. Fortunately, it can be canceled by using the offset-nulling circuit given by the manufacturer of the op-amp.

Hristo Stefanov is a consultant in Bulgaria. He specializes in hardware and software design for industrial automation and control applications. Hristo holds an M.S. in Engineering from Technical University of Plovdiv. He can be reached at hstefanov@mailandnews.com.

RESOURCES

P. Horowitz and W. Hill, *The Art of Electronics*, Cambridge University Press, New York, NY, 1993.

Analog Devices, "Ultraprecision Operational Amplifier OP177," http://products.analog.com/products/info.asp?product=OP177.

Vitrohm, "Competence in Electronics," 1995/96.

J. Graeme, "Op-amp Performance Analysis," AB-045, Burr-Brown Corp., 1993.

E. Nash, "Errors and Error Budget Analysis in Instrumentation Amplifier Applications," AN-539, Analog Devices.

Circuit Cellar, the Magazine for Computer Applications. Reprinted by permission. For subscription information, call (860) 875-2199, subscribe@circuitcellar.com or www.circuitcellar.com/subscribe.htm.